Indian Statistical Institute, Bangalore

M. Math First Year

First Semester - Measure Theoretic Probability

Backpaper Exam | Date: December 30, 2019

Maximum marks: 100 | Duration: 3 hours

Section I: Each question carries 10 Marks Total Marks: 100

1. Prove that the outer measure of an interval is the length of that interval.
2. Prove that the translate of a Lebesgue measurable set is Lebesgue measurable.
3. Let (μ_n) be a sequence of measures defined on a measurable space $(X, \mathcal{A})$. Suppose $\mu_n(E) \leq \mu_{n+1}(E)$ for any $E \in \mathcal{A}$. Prove that μ defined by $\mu(E) = \lim \mu_n(E)$ is a measure.
4. Prove the Lebesgue Dominated Convergence Theorem.
5. Let ν, μ and λ be σ-finite. If $\nu \ll \mu \ll \lambda$, prove that $\nu \ll \lambda$ and $\frac{d\nu}{d\lambda} = \frac{d\nu}{d\mu}\frac{d\mu}{d\lambda}$ a.e.
6. If m is the Lebesgue measure, find $m \times m(\{(x, y) \in \mathbb{R}^2 \mid yx^2 - xy^2 + xy - 7 = 0\})$.
7. Let X_n be random variables. Prove that $X_n \to 0$ in probability if and only if $E(\frac{|X_n|}{|X_n|+1}) \to 0$.
8. For a random variable X, prove that there is a m such that $P(X \leq m) \geq \frac{1}{2}$ and $P(X \geq m) \geq \frac{1}{2}$.
9. State and prove the central limit theorem.
10. For a $\mu \in M^1(\mathbb{R})$, determine when μ^n can converge.